EC 2020 Tutorial Exercises

Foundations of Non-truthful Mechanism Design

Part I: Equilibrium Analysis

1. Consider n bidders with values distributed i.i.d. from the uniform distribution on $[0, 1]$, identify the Bayes-Nash equilibrium of the first-price auction.

2. Consider two agents with values distributed i.i.d. from the uniform distribution on $[0, 1]$, there is a truthful mechanism that allocates to both agents when $\mathsf{v}_1 + \mathsf{v}_2 \geq 1$ and to neither of the agents, otherwise. Identify a winner-pays-bid mechanism that implements this allocation rule.

3. Consider two bidders with values uniform on [0,1] and [0,2], respectively, use revenue equivalence to prove that the first-price auction does not maximize welfare in equilibrium.

4. In the tutorial we proved that there are no asymmetric equilibria in the first-price auction with i.i.d. agent values in the case that the bid strategies cross at two points v' and v''. Prove that there are no asymmetric equilibria that cross at a single point v'. (You can assume that v' is strictly interior to the range of the bidders values.

5. Consider a $(1 + \epsilon)$ pure Nash equilibria, i.e., where each agent makes deterministic actions that obtain a utility that is within a multiplicative $(1+\epsilon)$ of a best response, in a deterministic mechanism that has conversion ratio μ. Prove that any such equilibria obtains a $(1 + \epsilon)\,\mu$ approximation to the optimal welfare. Note: in a pure Nash in a deterministic mechanism, the bid allocation rules take values 0 and 1.