EC 2020 Tutorial Exercises

Foundations of Non-truthful Mechanism Design

Part I: Equilibrium Analysis

1. Consider \( n \) bidders with values distributed i.i.d. from the uniform distribution on \([0, 1]\), identify the Bayes-Nash equilibrium of the first-price auction.

2. Consider two agents with values distributed i.i.d. from the uniform distribution on \([0, 1]\), there is a truthful mechanism that allocates to both agents when \( v_1 + v_2 \geq 1 \) and to neither of the agents, otherwise. Identify a winner-pays-bid mechanism that implements this allocation rule.

3. Consider two bidders with values uniform on \([0,1]\) and \([0,2]\), respectively, use revenue equivalence to prove that the first-price auction does not maximize welfare in equilibrium.

4. In the tutorial we proved that there are no asymmetric equilibria in the first-price auction with i.i.d. agent values in the case that the bid strategies cross at two points \( v' \) and \( v'' \). Prove that there are no asymmetric equilibria that cross at a single point \( v' \). (You can assume that \( v' \) is strictly interior to the range of the bidders values.

5. Consider a \((1 + \epsilon)\) pure Nash equilibria, i.e., where each agent makes deterministic actions that obtain a utility that is within a multiplicative \((1 + \epsilon)\) of a best response, in a deterministic mechanism that has conversion ratio \( \mu \). Prove that any such equilibria obtains a \((1 + \epsilon) \mu\) approximation to the optimal welfare. Note: in a pure Nash in a deterministic mechanism, the bid allocation rules take values 0 and 1.